SASTA OLIPHANT
SCIENCE AWARDS

Highly Commended

Scientific Inquiry

Year 11-12

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Scientific Inquiry (Logbook): How to maximise the "snowfall effect" in snowglobes?

Exploring the Problem: Manufacturers of snow-globes aim to make their product desirable by maximising the "Snowfall effect", which is achieved by increasing the time taken for flakes to reach the base after being shaken and achieve a state where flakes are almost floating in solution. Maximising this effect can be investigated through manipulating variables which alter forces acting on a spherical hollow bead in its descent to the base of a cylinder filled with glycerine, which is the most used liquid in snow-globes. Investigating time of descent can be achieved by considering forces acting on the snowflake which opposes its weight ($F_g = mg$), therefore decreasing net force downwards and magnitude of terminal velocity. As a result, both Drag force $\left(F_D = \frac{1}{2}\rho v^2 C_D A\right)$ and Buoyancy force ($F_B = \rho g V$) will be used to consider an independent variable to maximise the dependent variable of duration of fall of bead in solution. Finally, various methodologies will be proposed, where justifications will be used to determine the most appropriate procedure for measuring the "Snowfall" effect.

Brainstorm of Independent Variable:

Variable	Relevance To Duration of Fall	Appropriateness	Limitations
Temperature of Fluid	Temperature of fluid is inversely proportional to the density of the liquid $\left(T \propto \frac{1}{\rho}\right)$, as the kinetic energy of particles changes and therefore intermolecular spacing of particles that oppose beads motion changes. This is directly proportional to drag force ($\rho \propto F_D$) and the total duration to descend to base.	Temperature of solution can be incrementally increased through hot plates, where it is possible to find a quantifiable relationship between duration of fall and specific temperatures. Large range of temperatures can be tested, and accuracy of data can be validated through literature values of densities of solution at specific temperatures. Temperature can be precisely monitored by probes, limiting the presence of random errors.	Exposure to external surrounding may fluctuate the temperature exposed by the liquid, introducing potential random error. Heating bead may alter its kinetic energy, increasing its velocity and therefore introducing systematic errors on determining duration of fall. Heat may not be evenly distributed in the solution, therefore leading to random errors due to the non-uniform nature of particles intermolecular spacing.
Cross Sectional Area of Snowflakes	Radius of bead is directly proportional to drag force ($F_D \propto A$), as cross-sectional rea determines number of fluid particles opposing the beads motion, therefore changing time taken to reach base.	Possible to find a quantifiable relationship between effect of specific lengths and duration of fall. Cross-sectional area of snowflake can be determined through measuring its diameter.	Surface area of irregular structures can be hard to determine and therefore affect reliability of data when determining true effects from drag. Differences in design also alter the drag coefficient, providing uncontrolled error that limits the determination of effect of only cross-sectional surface area on duration of fall.
Height Dropped from Above Cylinder	Changing drop height changes the initial velocity of the ball (v_0), therefore affecting time to reach terminal velocity and changing overall duration of descent.	Initial drop height can be easily manipulated by incrementally changing the height of bead dropped above the cylinder. Same fluid and materials can be used when repeating trials, limiting the effect of random error when using different materials. Possible to find quantifiable relationship with drop height and time.	Human error may result in inconsistencies when dropping from height above cylinder, therefore introducing random errors. Changes in initial velocity of solution upon entering solution would introduce systematic error for time to descend to base. Has limited relevance to snow globes, as snowflakes descend to base when fully submerged in fluid.
Volume of Fluid	Volume of fluid is directly proportional to Buoyancy force ($F_B \propto V$) as the volume displaced changes. This affects the time to reach terminal velocity and therefore changes the total duration to descent.	Can be easily manipulated by testing different volumes of solution. Easily to repeat trials as through use of measuring equipment. Enables for a quantifiable relationship between volume of water and duration of fall. Trials are easy to repeat.	Extreme lengths of snowflake may drastically decrease the distance travelled by the snowflake. Has limited range when practically applying to the scale of snow-globes.
Density of fluid	Density of fluid is directly proportional to Buoyancy force ($\rho \propto F_B$), as the number of particles per unit volume opposing the beads descent changes, therefore changing the net force and time taken to reach base.	Concentration can be adjusted and easily repeated by changing the volume of Ethylene Glycol (antifreeze) present in mixture. Continuous range of data can be used to find quantifiable relationship with density and duration of fall. Density of solution can be accurately measured with Hydrometer and allows determination of ideal density.	Heterogenous mixture may be formed with Glycerine and Ethylene Glycol, therefore leading to non-uniform composition that may introduce random error during the flake's descent. Parallax error when measuring quantities of solution may be imprecise and lead to random errors when determining the effect of duration on the true volume.

Potential Methodology: The following methodologies were considered to measure the total duration of fall and time when bead reached terminal velocity.

Methodology	Potential Procedure	Advantages	Limitations
Stopwatches determining total time to reach base	After released from height, stop watches can be used to determine the total duration for snowflake to reach bottom of the flask. Given a constant distance, the trials can be repeated to mitigate the inconsistencies and random errors.	Stopwatches can be used to measure the total time taken for snowflakes to reach base. Procedure can be repeated for multiple attempts. Procedure is directly relevant to the dependent variable being total duration of fall.	Does not accurately indicate the velocity of the ball in the fluid and therefore the rate that it descends to the base. Use of manual recording can let the effects of human error be present, potentially leading imprecisions in time calculated and therefore presence of random error.
Slow-motion camera measuring time when reaching terminal velocity	Slow-motion cameras can be used to determine the time taken to travel set distances, where no change in time would indicate point of terminal velocity. Blank sheet with set distances can be set up behind measuring cylinder as bead falls.	Slow-motion cameras can be used to determine the time and distance for the ball to reach terminal velocity. Can be used to show the changes in velocity as bead descends, providing a more accurate indication of time to reach terminal velocity.	Is limited on its relevance for the total time to descended to base. Relies on human judgement to determine the time taken to travel set distances, decreasing the reliability of procedure due to presence of random error. Cannot precisely measure true velocity at points.
Light Gates positioned to measure time to travel set distances	Light gates can be placed at constant distance apart, measuring the time taken to travel set distances. As ball falls through light gates, it will determine the duration to travel set distance and display on data logger.	Light gates have a relatively low error-percentage, meaning it is very precise and limits the presence of random errors. They can be accurately placed at set distances and can be easily repeated for many trials without adjustments needed.	Does not provide indication of the velocity of the snowflake between the light gates, limiting the analysis in determining point where it reaches terminal velocity. Height dropped must be kept consistent, where additional velocity is not considered by the gates and therefore may introduce systematic errors.
Tracking software to measure distance travelled in set time periods	Recording of experiment can be processed into "Tracker" software which monitors the distance travelled and velocity of the spherical bead travelling in solution at any given period. Data can be logged on table and displayed on graph.	Technology can analyse recording in certain frames perf second, limiting presence of random error occurring and allow for instantaneous readings during procedure. Recording can be analysed many times, enabling more readings of data which can limit the presence of outliers.	Limited errors due to high precision in technology. Must be calibrated and processed through the aid of human when adjusting the parameters, which may introduce potential random errors.

Justification of Variable Investigated and Methodology:

Consideration of Independent Variable: Height above cylinder was eliminted based on its limited relevance towards Snowglobes which are fully filled containers. Volume of solution was similarly eliminated based on its effect on drop height and therefore inconsistencies in distance travelled by bead. Surface area of bead was also disregarded due to difficulty in manipulating and potential random error in irregular shapes. Temperature of solution was also eliminated based on its effect on velocity of ball and inconsistent effect on entire fluid that may introduce random error.

Overall Justification – Density of Solution: The ratio of Ethylene Glycol to Glycerine was chosen to investigate the effect of density on duration of fall. Density can be easily repeated with multiple trials and manipulated with precise measuring equipment. Density also has greater relevance to the application of snow-globes and enables the determination of an ideal ratio of a solution with constant volume that maximises the snowfall effect.

Consideration of Methodology: Light gates were not considered based on its inability to determine point of terminal velocity during beads descent and limitations in analysis for nature of motion between the gates. Similarly, Slow-motion camera was also disregarded due to its imprecision when measuring velocities at exact time intervals and reliance on human judgement that may decrease accuracy of experiment.

Overall Justification – Tracking Software/Stopwatches: It was chosen that computer tracking software would be used to measure velocity of bead at given time periods. Due to high precision and multiple frame rate, it is possible to determine the terminal velocity of bead and therefore provide a clear indication on effect of density for duration of fall. Stopwatches were also used in conjunction to measure the total time taken to descend the base, enabling the results to directly address the aim.

Proposed Apparatus (Justification in **Blue**):

- 2 250mL measuring cylinder (Acts as the "Snow-globe")
- 1 500mL measuring cylinder (Filters solution to retrieve bead for further trials)
- 1 Retort stand (Holds beads at consistent height to maintain initial velocity)
- 1 Boss heads (Same purpose as retort stand)
- 1 Clamp (Same purpose as retort stand)
- 1 30cm ruler (Measure height dropped above to maintain constant initial velocity)
- 100mL of 0.1M Glycerine (Used to measure independent variable)
- 500mL of 0.1M Ethylene Glycol (Used to measure the independent variable)
- 1 Stopwatch (Measure total duration of fall)
- 1 Tracker Software (Indicates time to reach terminal velocity and value)
- 1 Spherical bead with 11.5mm diameter (Acts as the "Snow-flake")
- 1 Mass Scale (Measure's mass of bead and therefore weight of ball)

Proposed Method (Justification in **Red**)

1. Apparatus was set up according to Figure 1. Mass and radius of bead were measured using mass scales and callipers respectively (measurements used for analysis)
2. 0.1M Glycerine and 0.1M of Ethylene Glycol was poured into 250mL measuring cylinder in a 1:0 ratio respectively, totalling a volume of 100mL. (100mL was used due to its validity when applying the experiment to real life snow-globes whilst also having enough time to measure velocities over greater time intervals)
3. Bead was placed 5cm above solution in measuring cylinder and suspended by clamps. Stopwatch and computer tracking system were positioned appropriately to monitor the trial (Constant height above solution ensured initial velocity was consistent)
4. Bead was released from metal clamp, where both stopwatches and computer recorded/monitored recorded the total time from beads release point until reaching bottom of cylinder (Technology measures total duration of descent and records the experiment when analysing the instantaneous velocity of bead respectively)
5. Results were recorded on table, where Glycerine was filtered into 500mL measuring cylinder to retrieve hollow bead and measuring cylinder was rinsed with distilled water. (Remove residue and impurities that may influence the density of solution. Hollow bead was retrieved to limit effect of random error through different bead size or shape)
6. Steps 2-5 were repeated for two more trials where the same equipment and solutions were utilised (Repetition of trials limits the presence of outliers and validates precision of experiment).
7. Steps 2-6 were repeated to test 1:1, 1:2, 1:3 and 1:4 ratios of 0.1M of Glycerine to 0.1M of Ethylene Glycol in 250mL measuring cylinder. Volume of Ethylene was determined through separate 250mL measuring cylinder (Used to investigate a wide range of densities, therefore providing reliable data when exploring the relationship between density and duration of fall)

Raw Data:

Volume of Distilled Water (mL)	Volume of Glycerine (mL)	Total Duration of Fall (seconds)			Average Time (3 Significant Figures)
		Trial 1	Trial 2	Trial 3	
100	0	1.62	1.34	1.31	1.42
90	10	1.55	1.81	1.91	1.76
80	20	1.80	1.75	1.60	1.72
70	30	1.90	1.94	2.05	1.96
60	40	2.49	2.34	2.47	2.43
50	50	2.50	2.75	3.10	2.78

Note: Volumes of Glycerine investigated were chosen to provide an adequate range that could indicate a trend for effect of density on duration of fall. However, range was not too broad such that the density of solution is greater than density of bead and therefore make the bead float. The trend could then be used to determine an ideal density of solution which maximises duration of fall. Through preliminary tests, it was decided that **distilled water** instead of **Ethylene Glycol** would be used in addition to Glycerine, as the high density of Ethylene Glycol meant that there would be marginal changes in duration of fall with different compositions and therefore invalid results. Furthermore, it was decided that **low percentage of Glycerine would be changed**, as test with 100% Glycerine caused the bead to maintain on top of solution and provided invalid results on how density effects duration of fall.

Risks/Ethical Considerations:

Consideration:	Significance and Mitigating Risks
Ingestion of Ethylene Glycol	Ethylene Glycol is a toxic substance that can lead to intoxication, central nervous system depression and gastrointestinal irritation if ingested. Appropriate personal equipment must be worn, and apparatus should be rinsed with distilled water after experimental trials. Incident of ingestion should be immediately notified to the supervisor.

References:

- Centres for Disease Control and Prevention (2018). *CDC - The Emergency Response Safety and Health Database: Systemic Agent: ETHYLENE GLYCOL - NIOSH*. [online] www.cdc.gov. Available at: https://www.cdc.gov/niosh/ershdb/emergencyresponsecard_29750031.html.

- Lumen Learning (2019). *Drag Forces | Physics*. [online] Lumenlearning.com. Available at: https://courses.lumenlearning.com/physics/chapter/5-2-drag-forces/.

How density of fluids in a snow globe impacts the “snowfall effect”

Introduction:

Background Theory:

Snow-globes function based on maximising the “snowfall effect”, which is achieved by increasing the duration of fall of a snowflake once shaken. Snowflakes motion in liquids can either be turbulent or laminar due to the ratio of inertial and viscous forces present[1]. With turbulent flow, snowflakes experience a Buoyant ($F_B = \rho V g$) and Drag ($F_D = \frac{1}{2} C_D A \rho v^2$) force which opposes the snowflakes weight $\left(F_g = mg\right)$ (Figure 1), where m is mass of object, A is cross-sectional area of snowflake, ρ is density of fluid, V is volume of fluid displaced, g is acceleration due to gravity, C_D is drag coefficient and v is velocity. Glycerine is the conventional liquid for snow-globes because its high density provides a greater force opposing the snowflakes weight and therefore increases the snowfall effect. However, Glycerine’s density may be larger than the snowflakes density such that the flakes do not sink. Therefore, manufactures alter the percentage of Glycerine in solution by adding distilled water, therefore changing the overall density of solution. When the snowflakes reach terminal velocity, it experiences no net acceleration as the forces balance out due to Newton’s First Laws of Motion. Therefore, a relationship for density and time can be derived at terminal velocity:

$$\text{As } F_g = F_D + F_B \text{ at Terminal Velocity}$$

$$\therefore mg = \frac{1}{2} C_D A \rho v^2 + \rho V g$$

$$mg = \rho\left(\frac{1}{2} C_D \pi r^2 v^2 + \frac{4}{3}\pi r^3 g\right)$$

$$\frac{s^2}{t^2} = \frac{2mg}{C_D \pi r^2 \rho} - \frac{8rg}{3C_D}, \therefore \frac{1}{t^2} = \frac{1}{\rho}K - C \ \left(\text{where } K = \frac{2mg}{C_D s^2 r^2 \pi}, C = \frac{8rg}{3C_D s^2} \text{ are constants}\right)$$

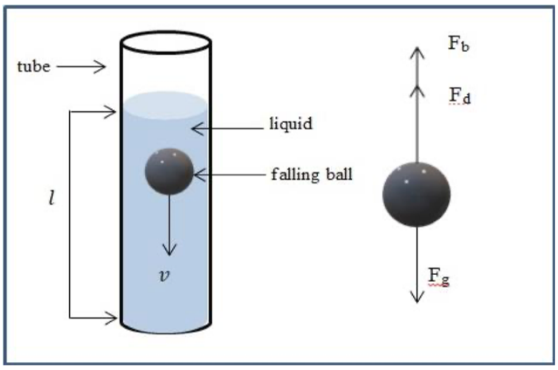


Figure 1: Forces acting on bead during descent to base

As the velocity of bead is inversely proportional to the time taken to fall a constant distance $\left(v \propto \frac{1}{t}\right)$, there is an expected linear relation between the inverse of density and inverse of duration of fall squared. Therefore, the experiment in question will investigate effects of varying ratios of Glycerine and distilled water on duration of fall.

Aim:

Investigate ideal density of fluid to maximise “snowfall effect” through measuring duration of fall of spherical bead in a 250mL measuring cylinder with different ratios of Glycerine to distilled water.

Hypothesis:

As ratio of Glycerine to distilled water increases in solution, then total duration of fall for the bead increases, as the density of solution increases and therefore proportionally increases Buoyant and Drag force acting against weight of the bead $(\rho \propto F_B \text{ and } \rho \propto F_D)$. As discussed in Background Theory, this decreases magnitude of terminal velocity and therefore increases duration of fall $\left(v \propto \frac{1}{t}\right)$.

Variables Table:

Type:	Significance:	How is it changed/measured/kept constant:
Independent: Ratio of Glycerine to distilled water	Ratio of Glycerine to distilled water in solution changes density of solution, therefore enabling investigation of the aim.	Tests with 0%, 10%, 20%, 30%, 40% and 50% of Glycerine in solution were conducted.
Dependent: Duration of Fall	Time taken to descend to base indicates the effect of density on snow-fall effect.	Total duration was measured using stopwatches and Tracker technology on computer
Controlled (1): Solution used	Inconsistencies in solution change overall density, changing net forces acting on bead and therefore introducing random error.	Glycerine and distilled water were used for all experimental trials.
Controlled (2): Volume of Solution	Volume of solution affects distance travelled (if investigated in the same container) leading to potential systematic error on duration of fall if not kept consistent.	100mL was maintained for all experimental trials and measured with same 250mL measuring cylinder
Controlled (3): Drop height of bead	Changes in drop height affect initial velocity of bead, affecting total duration of fall.	Bead was dropped 6cm above measuring cylinder. Height was measured using ruler and kept consistent using clamps.
Uncontrolled (1): Temperature	Changes in temperature affect density of solution ($T \propto \frac{1}{\rho}$), affecting net forces acting on bead.	All experimental trials were conducted under room temperature conditions.

Apparatus (Justifications in logbook):

- 2 x 250mL measuring cylinder
- 1 x 500mL measuring cylinder
- 1 x Retort Stand
- 2 x Boss heads
- 1 x Clamps
- 1 x 30cm ruler
- 1 x Stirring Rod
- 500mL of 0.1M Glycerine
- Distilled Water
- 1 x Stopwatch
- 1 x Tracker Software
- 1 x Spherical bead with radius 5.7mm and mass of 0.907g
- 1 x Mass Scales
- 1 x Callipers

Method (Justifications in logbook):

1. Apparatus was set up according to Figure 1. Mass and diameter of bead was measured using mass scales and callipers respectively.
2. Distilled water and Glycerine were poured into 250mL measuring cylinder in a 1:0 ratio respectively, totalling a volume of 100mL. Mass of solution was measured using mass scales.
3. Clamps were positioned approximately 6cm above solution. Stopwatch and computer tracking system were positioned to record the trial.
4. Bead was released from metal clamp, where both stopwatches and computer recorded/monitored the total time from beads release point until reaching bottom of cylinder.
5. Results were recorded, where solution was filtered into 500mL measuring cylinder to retrieve hollow bead and rinsed with distilled water.
6. Steps 2-5 were repeated for two more trials where the same equipment and solutions were utilised.
7. Steps 2-6 were repeated for tests of 10mL, 20mL, 30mL, 40mL and 50mL of 0.1M Glycerine in 250mL measuring cylinder. Total volume of solution remained 100mL and volume of Glycerine was determined through separate 250mL measuring cylinder.

Figure 2: Set up of Apparatus

Safety Risks:

Consideration	Significance and Mitigating Risks
Ingestion of Glycerine	Glycerine is a very toxic substance which if ingested can lead to intoxication, central nervous system depression and gastrointestinal irritation. Appropriate personal equipment must be worn, and apparatus should be rinsed with distilled water after experimental trials. Incidents of individuals ingesting liquid should be immediately notified to the supervisor.

Results:

Raw Data:

Table 1: Duration of Fall of Spherical Bead in Solution of Distilled Water and Glycerine

Volume of Distilled Water (mL)	Volume of Glycerine (mL)	Total Duration of Fall (seconds)			Standard Deviation (3 Significant Figures)	Average Time (3 Significant Figures)
		Trial 1	Trial 2	Trial 3		
100	0	1.62	1.34	1.31	0.171	1.42
90	10	1.55	1.81	1.91	0.186	1.76
80	20	1.80	1.75	1.60	0.104	1.72
70	30	1.90	1.94	2.05	0.0777	1.96
60	40	2.49	2.34	2.47	0.0814	2.43
50	50	2.50	2.75	3.10	0.301	2.78

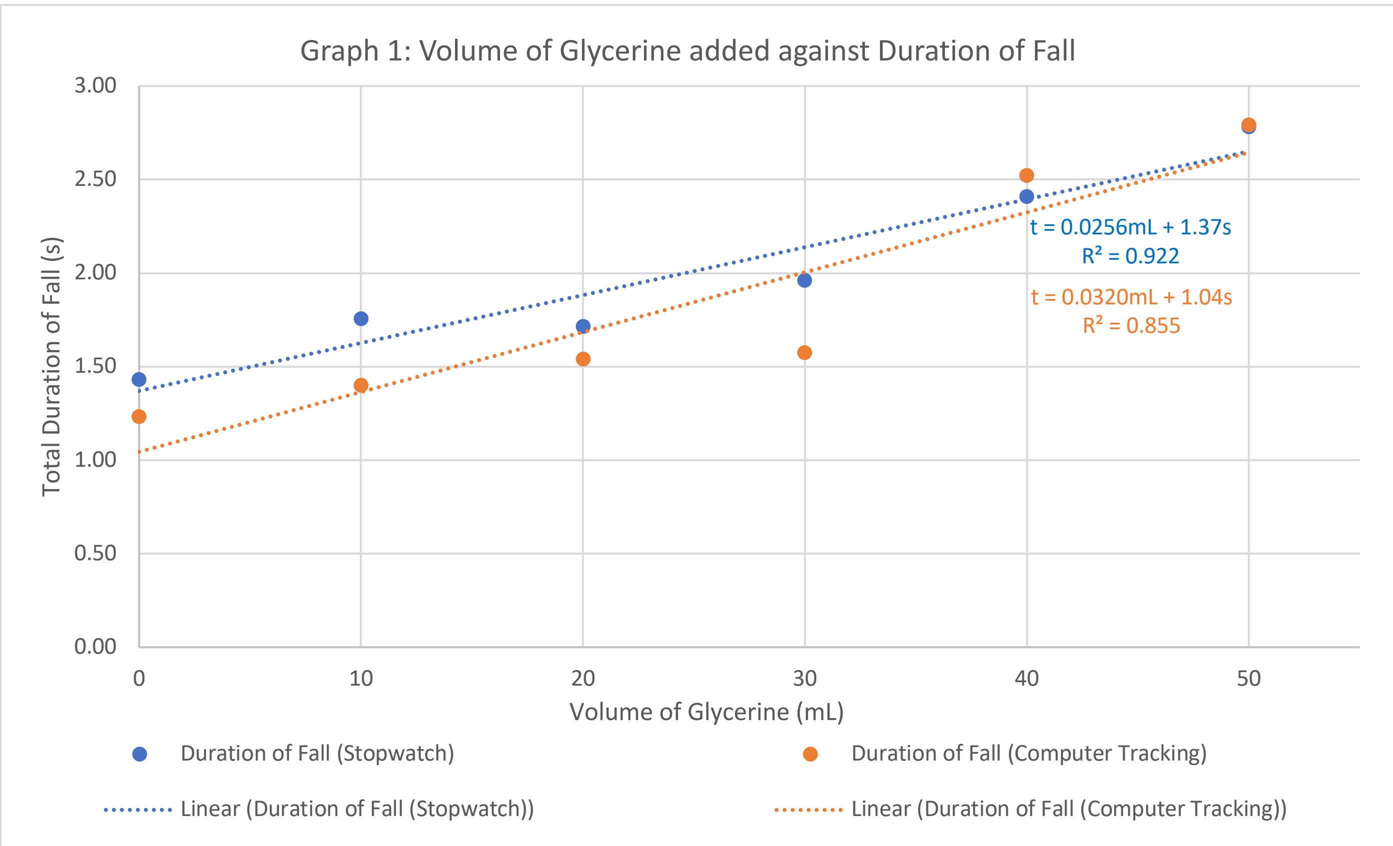
Table 2 shows calculations for the density of solution, which can be used to directly investigate the aim and analyse graphs. Mass of solution was determined using mass scales.

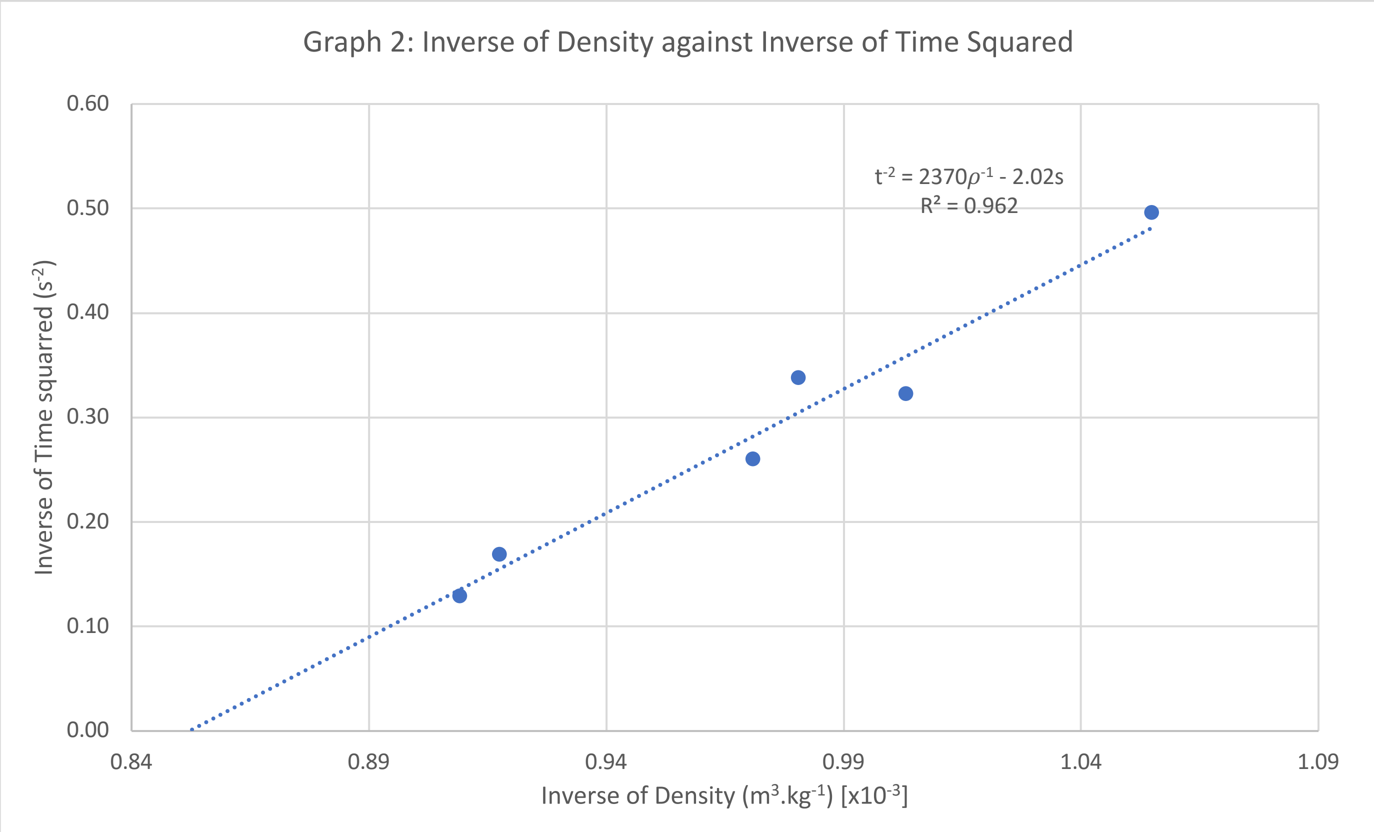
Table 2: Determining Density of Solution for Different Ratios of Distilled Water to Glycerine		
Percentage of Glycerine in 100mL Solution (%)	**Mass of Solution (kg) (3 Significant Figures)**	**Density of Solution (kg.m^{-3}) $\left[\rho = \frac{m}{v}\right]$**
0	0.0948	$\frac{0.0948}{0.1} \times 10^3 \approx 948$
10	0.0997	997
20	0.103	1030
30	0.103	1030
40	0.109	1090
50	0.112	1120

Table below shows results from tracking technology.

Table 3: Data from technology regarding duration of fall			
Percentage of Glycerine in 100mL Solution (%)	**Time (Seconds)**	**Value of R^2 from Graph**	**Standard Deviation**
0	1.233	0.970	0.00477
10	1.380	0.980	0.00356
20	1.529	0.969	0.00489
30	1.575	0.991	0.00256
40	2.525	0.987	0.00296
50	2.792	0.964	0.00465

Graphs:





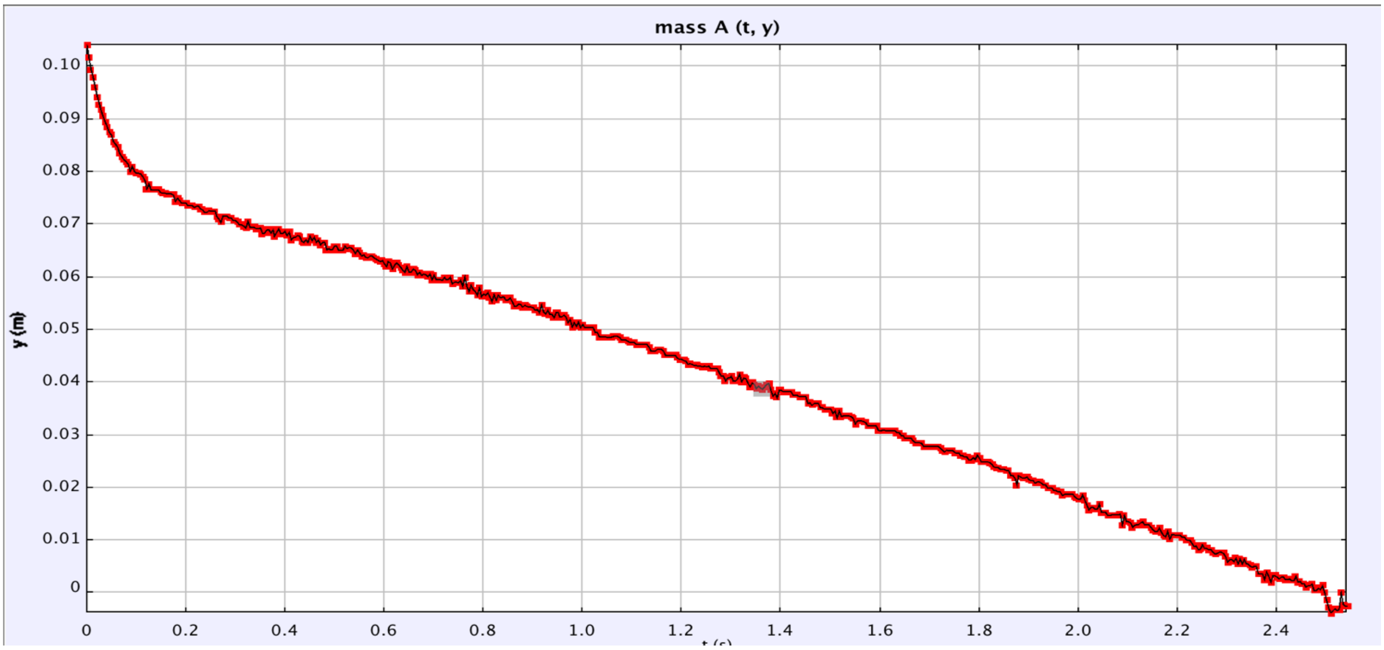


Figure 3: Graph above shows the displacement of the bead tracked across the duration of descent through tracker technology. Illustrates that displacement in first 0.2 seconds was not the same after 0.2 seconds.

Qualitative Observations

- Solutions had to be thoroughly stirred before experimental trials were conducted to ensure the solution was evenly distributed (relatively homogenous)
- Bead did not descend in solution at constant velocity, where it was observed to have greater velocity in the first milliseconds of descent (Figure 3 above).
- Bubbles were observed to emerge from hole of bead during descent.

Calculations:

Determining density of solution to maximise duration of fall:

Using experimental data, it is possible to find the density of solution which maximises duration of fall without floating in solution. Accuracy of experiment can be analysed through finding maximum density using the line of best fit and comparing with expected value.

$$\text{As } t \to \infty, \frac{1}{t^2} \to 0 \therefore \text{Using x} - \text{intercept of trendline in Graph 2:}$$

$$0 = \frac{2370}{\rho} - 2.02$$

$$\rho = \frac{2370}{2.02} \approx 1173.27 \text{ kg.m}^{-3}$$

Theoretical values can be determined through Archimedes principle, which state that objects float have a density that is less than the density of solution. Therefore, the maximum duration of fall can be found when the density of solution is equal to the density of the bead. Using $m = 9.07 \times 10^{-4}$ kg and $r = 0.0057$m:

$$\text{When } \rho_{Liquid} = \rho_{Bead}$$

$$\rho_{Liquid} = \frac{m}{v} = \frac{3(9.07 \times 10^{-4})}{4(0.0057)^3\pi} \approx 1169.21 \text{ kg.m}^{-3}$$

Therefore, the error-percentage of experimental data from theoretical data can be expressed below:

$$\frac{1169.21 - 1173.27}{1169.21} \approx 0.35\%$$

Determining model for duration of fall for the bead:

Using the equation established in the Background Theory for the net forces acting on the bead at any time, it is possible to derive a function that indicates its velocity at any time. From Newton's Second Law of Motion $(F = ma)$ and expressing $a = \frac{dv}{dt}$, the net force on the bead is (with positive direction being down):

$$F_{\text{Net}}: m\frac{\text{dv}}{\text{dt}} = mg - \frac{1}{2}C_D\rho Av^2 - \rho Vg$$

This expression can be simplified to:

$$a - bv^2 = \frac{\text{dv}}{\text{dt}}, \text{where } a = \frac{mg - \rho Vg}{m} \text{ and } b = \frac{C_D\rho A}{2m}$$

$$\frac{\text{dv}}{\left(v^2 - \frac{a}{b}\right)} = -b \text{ dt}$$

Using integration and separation of variables, it is possible to derive a function of velocity of the bead at any time. The definite integral has a lower bound of its initial velocity (v_0) and an upper bound of its velocity function $\big(v(t)\big)$. Technology was used to simplify the integration expression below[5]:

$$\int_{\text{v}'=\text{v}_0}^{\text{v}'=\text{v(t)}} \frac{\text{dv}'}{\left(v'^2 - \frac{a}{b}\right)} = -b\int_{\text{t}'=0}^{\text{t}'=\text{t}} dt'$$

$$\left(-\frac{\sqrt{b}\,\text{artanh}\left(\frac{\sqrt{b}}{\sqrt{a}}\,v(t)\right)}{\sqrt{a}} - \frac{\sqrt{b}\,\text{artanh}\left(\frac{\sqrt{b}}{\sqrt{a}}\,v_0\right)}{\sqrt{a}}\right) = -bt$$

$$\left(\frac{b}{\sqrt{b}}\sqrt{a}\right)t = \text{artanh}\left(\frac{\sqrt{b}}{\sqrt{a}}\,v(t)\right) + \text{artanh}\left(\frac{\sqrt{b}}{\sqrt{a}}\,v_0\right)$$

$$\sqrt{ab}t=\frac{1}{2}\ln\left(\frac{1+\sqrt{\frac{b}{a}}v(t)}{1-\sqrt{\frac{b}{a}}v(t)}\times\frac{1+\sqrt{\frac{b}{a}}v_0}{1-\sqrt{\frac{b}{a}}v_0}\right)$$

$$e^{2\sqrt{ab}t}=\frac{1+\sqrt{\frac{b}{a}}v(t)}{1-\sqrt{\frac{b}{a}}v(t)}\times\frac{1+\sqrt{\frac{b}{a}}v_0}{1-\sqrt{\frac{b}{a}}v_0}$$

As v_0 is the initial velocity and is constant, it is possible to let $\frac{1+\sqrt{\frac{b}{a}}v_0}{1-\sqrt{\frac{b}{a}}v_0}$ be expressed as a constant c.

$$\therefore \frac{e^{2\sqrt{ab}t}}{c}=\frac{1+\sqrt{\frac{b}{a}}v(t)}{1-\sqrt{\frac{b}{a}}v(t)}$$

$$e^{2\sqrt{ab}t}-e^{2\sqrt{ab}t}\left(\sqrt{\frac{b}{a}}v(t)\right)=c+c\left(\sqrt{\frac{b}{a}}v(t)\right)$$

$$\therefore v(t)=\frac{e^{2\sqrt{ab}t}-c}{\sqrt{\frac{b}{a}}\left(e^{2\sqrt{ab}t}+c\right)}$$

Therefore, for an initial velocity of v_0, the velocity of the bead during its descent at any point in time can be approximately calculated from the above equation. As drop height was 6cm above the solution and bead was released from rest, the velocity of the bead once in the solution can be shown below:

$$v^2=v_0^2+2as$$

$$\therefore v=\sqrt{2(9.81)(6.0\times10^{-2})}\approx 1.08\ \mathrm{m.s^{-1}}$$

Using the value v, it is possible to determine c in the function $v(t)$ and investigating different densities.

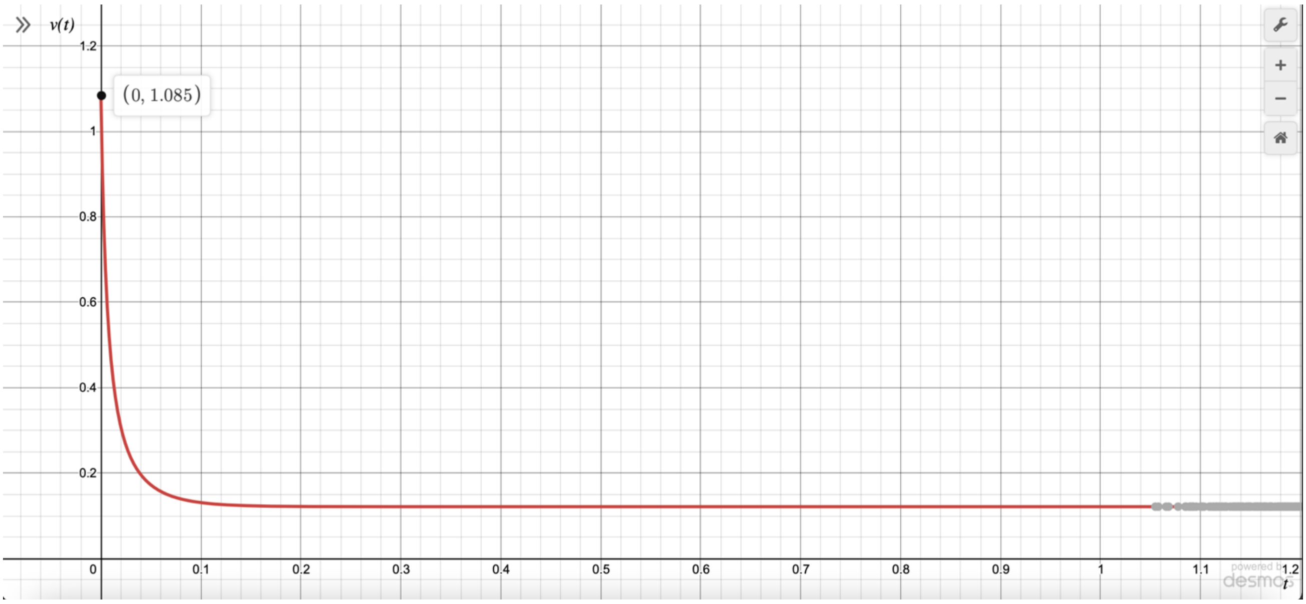


Figure 4: Graph of $v(t)$ for the bead with a density of fluid being 948 kg.m^{-3} (0mL of Glycerine) using theoretical calculations.

The graph for the velocity function can be compared with the graph from experimental data to discuss the accuracy of experiment:

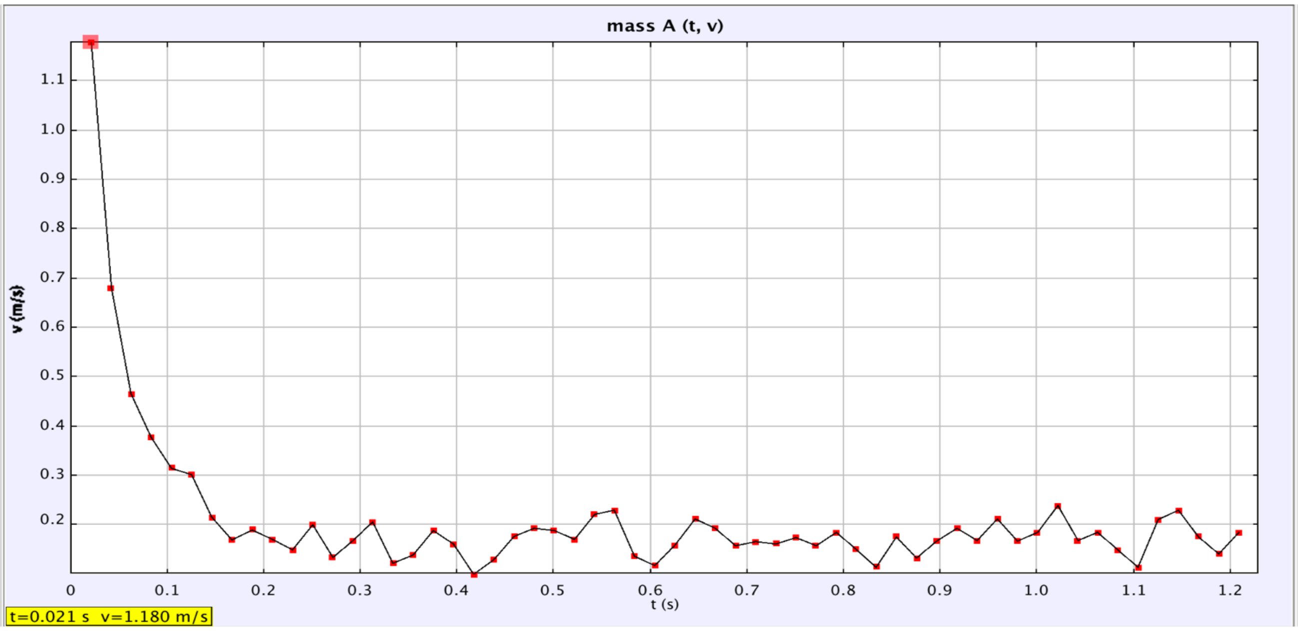


Figure 5: The $v(t)$ function of bead with a density of fluid being 948 kg.m^{-3} (0mL of Glycerine) from experimental data and tracking technology.

Therefore, through integrating $v(t)$, it is possible to determine the expected time taken to fall 25cm and therefore comment on the accuracy when comparing with experimental data;

Table 5: Comparing theoretical duration of fall from mathematical model with experimental data		
Volume of Glycerine	**Theoretical time of duration of fall.** $\left(\textbf{value of } t \textbf{ when } \int_0^t v(t)dt = 0.25\text{m}\right)$ **(3 significant figures)**	**Error Percentage** $\left(\% = \frac{\textbf{Experimental} - \textbf{Theoretical}}{\textbf{Theoretical}}\right)$
0	1.954	$\frac{1.954 - 1.233}{1.954} \approx 36.9\%$
10	2.267	39.1%
20	2.558	40.2%
30	2.558	38.4%
40	3.471	27.3%
50	4.441	37.1%

Discussion:

Results from experimental trials supported the hypothesis, as Graph 1 showed increasing percentage of Glycerine in solution increased duration of fall. Graph 2 indicated a positive linear trend between inverse of density and inverse of time squared, affirming the background theory. However, there is insufficient evidence to support the model when extrapolating from the trendline, where the large density of solution could mean the buoyancy force exceeded the weight of the object, where the bead remained on top of solution and therefore would not support hypothesis.

Experimental data indicates relatively high precision when measuring duration of fall, where high correlation coefficient in Graph 1 (R^2 = 0.922) and Graph 2 (R^2 = 0.962) suggests stopwatches were relatively precise in determining duration of fall. Repeating trials three times ensured outliers could be mitigated, especially from the large standard deviation (ranging from 0.07 to 0.31). Using technology enabled a more precise reading of duration of fall, however, hold limitations as values were only based on the first trial. Discrepancies between times from stopwatch and technology can be justified, as times from stopwatch were determined from moment of release whilst tracker technology determined time whilst in solution. However, significant scatter in both Graph 1 and Graph 2 suggests limitations upon measuring different densities, where Graph 2 did not show density increasing incrementally. Presence of outliers further suggests precision error, as duration of fall for 20mL of Glycerine was expected to be greater than 10mL and implies calculation for duration of fall was underestimated.

Experimental data was relatively accurate when modelling duration of fall with different densities, as Graph 2 affirmed the model in background theory. Comparison with the mathematical model indicated a relatively small error percentage for each trial (ranging from 27.3% - 40.2%), indicating the experiment outlined expected motion of bead in solution and therefore the methodology was relatively accurate. However, the drop height meant the bead travelled in solution with initial velocity, providing a systematic error that affects duration of fall. Data was relatively valid when finding ideal density of fluid to maximise duration of fall, as interpolation of trendline indicated 1163 $kg.m^{-3}$, with an 0.3% error percentage when comparing with expected values.

Evaluation:

Systematic – Height bead was dropped from

Dropping bead 6cm above solution meant it entered with initial velocity that decreased duration of fall. Change in momentum upon impact with solution resulted in force being applied $(F \propto \Delta p)$ that would affect beads motion. The consistent launch height for all trials meant duration of fall would consistently be an underestimate. This is evident in Figure 3 and qualitative observations, showing bead travelled greater distance upon entry and therefore decreased duration of fall. As snowflakes descend inside solution of snow globe, drop height would be expected to have significant effects on validity of experiment for snow globes. Improvements would be to change the methodology such that the bead was initially in solution, and it would be possible to precisely determine duration of fall.

Random/Systematic – Hollow nature of bead

Observations indicated bubbles forming during beads descent, suggesting the molecules opposing the beads motion varying from different orientations and therefore changes in drag force. Whilst force experienced inconsistently varies with orientation, the negative linear correlation between terminal velocity and drag cross-sectional area meant changes in orientation would constantly under-estimate duration of fall. This is evident in large standard deviation within trials, where discrepancies in time could be due to changes in orientation. As the hole prevents the determination of value of drag coefficient and therefore theoretical duration of fall, this error would have significant effect on accuracy of experimental data and validity for snow-globes. Improvement for this error can be minimised through utilising a manufactured snowflake.

Random **– Reaction time of stopwatch**

The uncontrolled nature of the observer's reaction time would result in an under or overestimate for duration of fall. This is evident in the large standard deviation and scatter within trials, where inconsistencies of time affected the values plotted. The outlier for 10 and 20mL of Glycerine indicated the reaction time having a significant effect of precision of experiment. Millisecond difference between different densities meant reaction time would have a significant effect on the trend of data and therefore the overall reliability for investigating snow-fall effect. Improvement to this error would be to use light gates to obtain precise value for duration of fall.

Conclusion:

Experimental data supported the hypothesis, as increasing percentage of Glycerine in solution with distilled water increased duration of fall as shown in trendline of Graph 2. The methodology was relatively reliable upon investigating the aim, as high precision suggested an ideal density of 1173 $kg.m^{-3}$ to maximise "snowfall effect". However, the bead investigated and the distance travelled may not be valid in application for snow-globes, as these conditions are not conventional for snow globes. Uncertainty in affirming experimental data with expected values and limited range of densities tested, requires extensions to methodology to test ideal density for snow-globes.

Word Count: 2170

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OSA RISK ASSESSMENT FORM

for all entries in (✓) ☐ Models & Inventions and ☑ Scientific Inquiry

This must be included with your report, log book or entry. One form per entry.

NAME: Dinan Perera ID: 0526-002

SCHOOL: Prince Alfred College

Activity: Give a brief outline of what you are planning to do.

The experiment will investigate how different densities of fluid effect the duration of fall of a spherical bead. It will be conducted by mixing Glycerine and distilled water in a measuring cylinder and determining the time taken for the bead to reach the bottom of the solution. Differing densities will be tested by changing the ratio of Glycerine to distilled water.

Are there possible risks? Consider the following:

- Chemical risks: Are you using chemicals? If so, check with your teacher that any chemicals to be used are on the approved list for schools. Check the safety requirements for their use, such as eye protection and eyewash facilities, availability of running water, use of gloves, a well-ventilated area or fume cupboard.
- Thermal risks: Are you heating things? Could you be burnt?
- Biological risks: Are you working with micro-organisms such as mould and bacteria?
- Sharps risks: Are you cutting things, and is there a risk of injury from sharp objects?
- Electrical risks: Are you using mains (240 volt) electricity? How will you make sure that this is safe? Could you use a battery instead?
- Radiation risks: Does your entry use potentially harmful radiation such as UV or lasers?
- Other hazards.

Also, if you are using other people as subjects in an investigation you must get them to sign a note consenting to be part of your experiment.

Risks	How I will control/manage the risk
- Danger of ingesting Glycerine - Broken Glassware (potential cuts and abrasions)	- Thorough rinsing of measuring cylinder after each trial - Washing of hands and desktop - Ensuring setup is away from edges - Wearing personal safety equipment (goggles and apron) - Notify relevant supervisor if accident has occurred

(Attach another sheet if needed.)

Risk Assessment indicates that this activity can be safely carried out

RISK ASSESSMENT COMPLETED BY (student name(s)): Dinan Perera

SIGNATURE(S): DINAN

☑ By ticking this box, I/we state that my/our project adheres to the listed criteria for this Category.

TEACHER'S NAME: Daniel Kerrigan

SIGNATURE: DATE: 23 – 7 – 21